

Решение дробных рациональных уравнений



$$1) \frac{x-1}{2} + \frac{2x}{3} = \frac{5x}{6}$$

$$\frac{x-1}{2} + \frac{2x}{3} = \frac{5x}{6} / \cdot 6$$

$$3(x-1) + 2 \cdot 2x = 5x$$

$$3x - 3 + 4x = 5x$$

$$3x + 4x - 5x = 3$$

$$2x = 3$$

$$x = 1,5$$

Ответ : 1,5

$$2) \frac{2x}{x-3} + \frac{11}{2} = \frac{3}{x}$$

$$\frac{2x}{x-3} + \frac{11}{2} = \frac{3}{x} / \cdot 2x(x-3) \neq 0$$

$x \neq 0; x \neq 3$

$$2x \cdot 2x + 11x(x-3) = 3 \cdot 2(x-3)$$

$$4x^2 + 11x^2 - 33x = 6x - 18$$

$$4x^2 + 11x^2 - 33x - 6x + 18 = 0$$

$$5x^2 - 13x + 6 = 0$$

$$a = 5; b = -13; c = 6$$

$$5x^2 - 13x + 6 = 0$$

$$a = 5; b = -13; c = 6$$

$$D = b^2 - 4ac = 169 - 120 = 49 > 0 (2\kappa)$$

$$x_{1;2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{13 \pm 7}{10}$$

$$x_1 = 2$$

$$x_2 = 0,6$$

Ответ : 2; 0,6

$$3) \frac{2}{2-x} + \frac{1}{2} = \frac{4}{2x-x^2}$$

$$\frac{2}{2-x} + \frac{1}{2} = \frac{4}{x(2-x)}$$

$$\frac{2}{2-x} + \frac{1}{2} = \frac{4}{x(2-x)} \quad / 2x(2-x) \neq 0$$

$x \neq 0; x \neq 2$

$$2 \cdot 2x + x(2-x) = 8$$

$$4x + 2x - x^2 - 8 = 0$$

$$-x^2 + 6x - 8 = 0$$

$$-x^2 + 6x - 8 = 0$$

$$a = -1; b = 6; c = -8; k = 3$$

$$\frac{D}{4} = k^2 - ac = 9 - 8 = 1 > 0 (2k)$$

$$x_{1;2} = \frac{-k \pm \sqrt{\frac{D}{4}}}{a} = \frac{-3 \pm 1}{-1}$$

$$x_1 = 4$$

$x_2 = 2$ — не является корнем уравнения

Ответ : 4

$$4) \frac{2}{x^2 - 4} - \frac{1}{x^2 - 2x} = \frac{4 - x}{x^2 + 2x}$$

$$\frac{2}{(x - 2)(x + 2)} - \frac{1}{x(x - 2)} = \frac{4 - x}{x(x + 2)} \quad /x(x - 2)(x + 2) \neq 0$$

$x \neq 0; x \neq -2; x \neq 2$

$$2x - (x + 2) = (4 - x)(x - 2)$$

$$2x - x - 2 = 4x - 8 - x^2 + 2x$$

$$2x - x - 2 - 4x + 8 + x^2 - 2x = 0$$

$$x^2 - 5x + 6 = 0$$

$$a = 1; b = -5; c = 6$$

$$D = b^2 - 4ac = 25 - 24 = 1 > 0 (2\kappa)$$

$$x_{1;2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{5 \pm 1}{2}$$

$$x_1 = 3$$

$x_2 = 2$ — не является корнем уравнения

Ответ: 3

$$5) * \frac{1}{x^2 + 3x - 3} + \frac{2}{x^2 + 3x + 1} = \frac{7}{5}$$

пусть $x^2 + 3x = m$

$$\frac{1}{m-3} + \frac{2}{m+1} = \frac{7}{5} \quad / 5(m-3)(m+1) \neq 0$$

$m \neq 3; m \neq -1$

$$5(m+1) + 2 \cdot 5(m-3) = 7(m-3)(m+1)$$

$$5m + 5 + 10m - 30 = 7(m^2 + m - 3m - 3)$$

$$5m + 5 + 10m - 30 - 7m^2 - 7m + 21m + 21 = 0$$

$$-7m^2 + 29m - 4 = 0$$

$$-7m^2 + 29m - 4 = 0$$

$$a = -7; b = 29; c = -4$$

$$D = b^2 - 4ac = 841 - 112 = 729 > 0 (2\kappa)$$

$$m_{1;2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-29 \pm 27}{-14}$$

$$m_1 = 4$$

$$m_2 = \frac{1}{7}$$

1) если $m = 4$, то $x^2 + 3x = 4$
 $x^2 + 3x - 4 = 0$

$$x^2 + 3x - 4 = 0$$

$$a = 1; b = 3; c = -4$$

$$D = b^2 - 4ac = 9 + 16 = 25 > 0 (2\kappa)$$

$$x_{1;2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm 5}{2}$$

$$x_1 = -4$$

$$x_2 = 1$$

2) если $m = \frac{1}{7}$, то $x^2 + 3x = \frac{1}{7}$

$$x^2 + 3x - \frac{1}{7} = 0 / \cdot 7$$

$$7x^2 + 21x - 1 = 0$$

$$7x^2 + 21x - 1 = 0$$

$$a = 7; b = 21; c = -1$$

$$D = b^2 - 4ac = 441 + 28 = 469 > 0 (2\kappa)$$

$$x_{1;2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-21 \pm \sqrt{469}}{14}$$

$$\text{Ответ : } -4; 1; \frac{-21 \pm \sqrt{469}}{14}$$